

The Numerical Solution of Reduced Differential Algebraic Equations

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A reduced differential equation results from applying any reduction procedure to a differential equation. We do not follow the often applied dummy derivative with projection technique to numerically solve the reduced differential equation. Instead we investigate a geometric construction.

A Rabier and Rheinboldt reduced differential equation leads to a tangent vector field on the core manifold. We show the strangeness reduction of Kunkel and Mehrmann as well as the sigma method of Pryce also lead to a tangent vector field on the core manifold.

Modern approaches to numerically solve an ODE on a manifold (ODEM) include stabilization techniques to the invariant manifold and projection techniques. We construct and solve the local ODE to solve the ODEM applying Rheinboldt's algorithms to evaluate local parametrizations.

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